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Properties of Supermembrane Axions

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Abstract. In the presence of membranes, M-theory becomes in the low energy limit 11 dimensional supergravity action coupled to a supermembrane action. The fields of the first action are the same fields which couple to the membrane. It is shown that the axionic moduli of the membrane obtained by wrapping the three form potential about three-cycles of a Calabi-Yau manifold can take nonzero integer values. This novel property allows M-theory to have smooth transition from the Kahler cone of a geometrical phase to a Kahler cone of another geometrical phase. Nongeometrical phases which define the boundary of the extended Kahler cone of the geometrical phases have discrete spectrum, and are continuously connected to the geometric phases. Using this new property, we relate the M-theory model dependent axion to the type IIA model dependent axion and show that a potential develops for the type IIA axion in the strong coupling regime which does not seem to be generated by instantons. Evidence is presented, using these moduli, which supports the Strominger conjecture on the winding p-branes.

1 Introduction

It is by now accepted that M-theory has supermembranes in its spectrum, and that it has as its low energy limit 11 dimensional supergravity. The low energy limit of M-theory in the presence of membranes is then 11 dimensional supergravity coupled to a supermembrane action. Moreover, the coupling of the supermembrane to this supergravity action implies that the fields which appear in the supermembrane action are the same as those in the supergravity action.

Three issues of M-theory will be dealt with in this paper under this assumption. The first one regards the phases of M-theory in the absence of membranes [1] analyzed by Witten. The second one regards the generation of a non perturbative potential for the type IIA axion in the strong coupling limit. The third one regards the Strominger conjecture on winding p-branes [2].

Phases

M-theory on a Calabi-Yau manifold [3] can have different phases [1]. In order to analyze them, it is necessary to define the Kahler cone of a given Calabi-Yau manifold X as the space of all Kahler metrics of X . This cone has a boundary where the size of a rational curve shrinks to zero size. It is also possible to move to the exterior of the Kahler cone: a curve defined on the manifold X will have negative volume in this region, but it is possible to flop the rational curve so that its volume becomes positive when interpreted as a rational curve of a manifold Y which is birationally equivalent to X . Then, by varying the moduli of the Kahler cone, we can transition to the set of manifolds which are birationally equivalent to the manifold X , and it is thus convenient to define the extended Kahler cone, as the union of all Kahler cones of the manifolds which are birationally equivalent to the manifold X and which lead to geometrical phases.

In [1], the behavior of 11 dimensional supergravity on the boundary of a Kahler

cone not lying on the boundary of the extended Kahler cone was analyzed. It was shown that the 5-dimensional Chern-Simons term signals when a transition from X to Y takes place. At the classical level, this phase transition is sharp since the spectrum becomes continuous. Also, it was argued that, as opposed to string theory where one finds abstract phases outside the extended Kahler cone (such as Landau-Ginzburg orbifold), in 11 dimensional supergravity the moduli space seems to end abruptly, and abstract phases are absent. In order to reach these conclusions, it is necessary to make a detailed analysis of the Kahler moduli space. We will see in the next sections by coupling the supermembrane to 11 dimensional supergravity and including the moduli space of three-cycles in the analysis of the phases actually smoothes out all transitions of M-theory including those which are non geometric phases.

Axionic Potential

The moduli of these three-cycles in M-theory are periodic but as we shall show take integer values. This is to be contrasted with the moduli of string theory obtained by wrapping the B-field about two-cycles. The latter moduli are periodic but do not take integral values. After compactification of M-theory on $S^1 \times X \times R^4$, it is possible to relate the moduli of three-cycles in M-theory to the moduli of two cycles in type IIA string theory on $X \times R^4$. The latter are model dependent axions of the type IIA string. In the strong coupling limit, the type IIA axions become M-theory axions. This means that as the coupling constant increases, the type IIA axions are restricted to certain discrete values which implies the existence of a potential. This potential does not seem to be generated by instanton effects, and at present, it is not possible to estimate the order of magnitude of the mass acquired by the axions.

The Winding Conjecture

In order to interpret the divergent coupling of string theory near a conifold singularity, Strominger [2] postulated that the winding of p-branes with $p > 1$ should

be treated in a different manner than the winding of strings. We must require that a p-branes with $p > 1$, winding $N > 1$ times be treated as an N particle state. Otherwise, a p-brane which wraps N times about a p-cycle of a compactification would contribute to the beta function in a similar way as a p-brane wrapped once about the same cycle. This, of course, ruins the relation between the conifold singularity and the renormalization of the coupling. On the other hand, as explained in [4], a string winding $N > 1$ times about a cycle must be treated as a single particle state rather than a multiple particle state because there are bound states at threshold [5].

In the context of string theory and without any knowledge of M-theory, discriminating between the winding properties of type IIA membranes and the type IIA strings does not pose a problem. But as soon as M-theory comes into the picture, some inconsistencies appear. For example, a membrane wrapped once on S^1 yields after dimensional reduction a type IIA string [6]. But what kind of string will a membrane wrapped n times about an S^1 yields? Given Strominger's conjecture, we expect that the string obtained does not depend on the number of times the membrane is wrapped about the S^1 . This is equivalent to the requirement that the string coupling and the axionic charge should not depend on the winding number of the membrane about S^1 . However, consider a membrane which is wrapped n_1 times about the first one-cycle of T^2 and n_2 times about the second one-cycle of T^2 . If we use the rule that dimensional reduction of a membrane yields the same string, then dimensional reduction of the membrane about the first one cycle yields the statement that N windings of the string about the second cycle should lead to an N particles state. This is then in contradiction with the departing conjecture that N windings of a string about a one-cycle should yield a single particle state. Equivalently, it is in contradiction with the findings of [5]: an n -wound string should not yield the same state as n single-wound string.

We will later see that the coupling (M-theory axion moduli) of the membrane to the three form potential C has a different structure than the coupling (type IIA

axion moduli) of the string to the two form potential B . This difference between M-theory and type IIA axionic moduli leads to a resolution of the Strominger winding conjecture.

Summary and Outline

Although we do not have a good grip on the quantum field theory describing the fundamental membrane, recent results point out that the supermembrane action will be quantized [7]. Also, supermembranes seem to be reasonable candidates whose low energy limit yields 11-dimensional supergravity [8].

Here we assume that M-theory in its low energy limit is 11 dimensional supergravity coupled to membranes. We then analyze the phases of supermembranes propagating on a Calabi-Yau threefold to draw conclusions on the phases of M-theory. We will restrict to Calabi-Yau three-folds, although higher dimensional compactification have also lead to new and interesting physics in lower dimensions [9, 10]. Supermembranes propagating on a Calabi-Yau threefold compactification have two types of moduli. The first type are those obtained by integrating the Kahler metric over a two-cycle of the compactification. These will not play a very important role in our discussion and most of their aspects have been dealt with in [3, 1]. The second type is obtained by wrapping the three-form potential about a three-cycle of the internal space. These moduli will be our *vedette*. By analyzing the gauged linear sigma model with N=2 supersymmetry in three dimensions, we will be able to conclude that these moduli take discrete values. These property can be used to show that in the presence of membranes, the transition of M-theory from one manifold X to another manifold Y takes place in a smooth manner. We will also study the phase transition at the boundary of the extended Kahler cone and show that non geometrical phases such as Landau Ginzburg orbifolds, having a discrete spectrum, are present at the boundary of the extended Kahler cone, and are continuously connected to this cone when membranes are present. In addition we will use this novel feature of supermembranes to

show the existence of an axionic potential in the strong coupling limit and to provide evidence in favor of the Strominger conjecture.

The paper is organized as follows. In section 2 we review the string gauged linear sigma model to explain which are the quantum effects which allow for smooth phase transitions between strings propagating on birrationally equivalent manifolds. In Section 3 we construct the membrane gauged linear sigma model coupled to 11 dimensional supergravity and show that quantum effects, absent in the low energy effective theory, also allow for the smooth phase transitions of M-theory propagating on birrationally equivalent manifolds. Section 4 deals with the analysis of phase transitions between geometrical and nongeometrical phases of M-theory theory. In Section 5 we show the existence of an axionic potential in the strong coupling limit of string theory. The last section is devoted to providing evidence for the Strominger winding conjecture.

2 Phases of N=2 Strings

We begin by reviewing the string gauged linear sigma model [11] used to describe the phase transitions of string theories with N=2 supersymmetry between two birrationally equivalent manifolds. The two dimensional Lagrangian is obtained from the dimensional reduction to D=2 N=2 of a D=4 N=1 gauge action coupled to matter [11]. The bosonic sector is¹

$$\begin{aligned}
L &= L_{kin} + L_{gauge} + L_D \\
L_{kin} &= \int d^2y (\partial_m \bar{a}_i \partial^m a_i + \partial_m \bar{b}_i \partial^m b_i) \\
L_{gauge} &= \int d^2y (v_{01}^2 + \partial_m \sigma \partial^m \bar{\sigma}) \\
L_D &= -r \int d^2y D.
\end{aligned} \tag{1}$$

¹We follow the conventions of [11]

L_{kin} is the kinetic term for four chiral superfields A_i, B_i ; $i = 1, 2$, whose bosonic scalars are a_i and b_i respectively. L_{gauge} is the kinetic term for the gauge field v_m $m = 0, 1$ (giving rise to the field strength v_{01}) and $\sigma = \sigma_1 + i\sigma_2$, where σ_i are the scalars of the dimensionally reduced D=4 vector multiplet. Thus, v_m and σ_i make up part of a D=2, N=2 vector multiplet. L_D is the Fayet Illiopoulos (FI) term and D is the auxiliary field of the D=2 vector multiplet. In addition, we may add a topological term

$$L_\theta = \theta \int d^2y v_{01} \quad (2)$$

which will be crucial in our discussion. The parameter θ is just the two dimensional theta-angle which is periodic [13]

$$\theta \sim \theta + 1. \quad (3)$$

It is the expectation value of the 4-dimensional model dependent axion field.

We will consider the example of [11] where the A_i 's will carry positive unit charge with respect to the gauge field and the B_i 's will carry negative unit charge. This complex fields can be thought as coordinates on C^4 . The bosonic potential of this model is obtained by integrating out the auxiliary field D

$$U = \frac{e^2}{2} (\sum_i |a_i|^2 - |b_i|^2 - r)^2 + |\sigma|^2 (\sum_i |a_i|^2 + |b_i|^2). \quad (4)$$

For $r \gg 0$, the a_i 's cannot both be zero since the first term in the bosonic potential is proportional to D^2 . The values of a_i determine a point in a copy of CP_a^1 . For $b_i = 0$, the gauge symmetry can be used to divide $\sum_i |a_i|^2 = r$ by $U(1)$ and therefore, r is the Kahler form of CP_a^1 . The zero section of the symplectic quotient, Z_+ , $Z_+ \rightarrow CP_a^1$, is a genus zero holomorphic curve which is obtained by setting $b_i = 0$ [11].

For $r \ll 0$, the role of the a_i 's and the b_i 's are exchange, and the b_i 's define a point in CP_b^1 with Kahler form $-r$. The zero section of the symplectic quotient Z_- is also a genus zero holomorphic curve.

As we transition from positive to negative r a change of topology, a flop, takes place because the size of CP_a^1 shrinks to zero size and is replaced by CP_b^1 . This means that Z_+ is replaced by Z_- and therefore, we move from a manifold X where CP_a^1 has positive volume to a birationally equivalent manifold Y where CP_b^1 has positive volume. In order to arrive to the point $r \ll 0$, we must first transverse the point $r = 0$. At this point in the moduli space, there are wave functions supported near $b_i = a_i = 0$ and $|\sigma| \gg 0$ which have a continuous spectrum. However, when θ in the topological term (2) is different from zero, the spectrum remains discrete, thus insuring a smooth phase transition [11].

The term (2) can be combined with the FI term

$$it \int d^2y (D + iv_{01}) \quad (5)$$

where $t = \theta + ir$ is the moduli of the complexified metric $J = B + iG$ where B is the antisymmetric tensor and G is the Kahler metric. Thus, θ is the coupling of the antisymmetric tensor. This also follows from the fact that for $r \gg 0$ we may rewrite (2) in the form

$$\theta \int d^2y \epsilon^{ij} \partial_i X^M \partial_j X^N B_{MN} \quad (6)$$

where $B_{MN} = \partial_{[N} A_{M]}$ is the antisymmetric tensor which locally is pure gauge, and the fields X^M parametrize a 10 dimensional space.

3 Phase Transitions Between Geometric Phases of M-theory

Is this transition between birationally equivalent manifold also smooth for membranes? If so, this implies that in the precence of supermembranes the transitions of M-theory are smooth.

The parameters r and θ in string theory are scalars of a D=4 N=2 spacetime vector multiplet. In the case of 11 dimensional supergravity on a manifold X , the N=2

vector multiplet is five dimensional, and therefore has one scalar only. This effectively sets θ to zero, thereby suggesting only the presence of sharp phase transitions. However, as we shall see, when supermembranes are added to the 11 dimensional supergravity action, there are moduli parametrizing the coupling of the three-form antisymmetric tensor C_{MNP} . These moduli allow the phase transitions of M-theory between birrationally equivalent manifolds to be smooth, just like in string theory.

Consider the dimensional reduction to D=3 N=2 of the N=1 D=4 gauge action coupled to matter. The dimensional reduction of this theory down to D=2 will yield the string gauged linear sigma model. This will guarantee that the model describes supermembranes whose dimensional reduction on S^1 yields strings. The D=3 N=2 action will yield the membrane gauged linear sigma model. The bosonic sector of the Lagrangian is

$$\begin{aligned}
L &= L_{kin} + L_{gauge} + L_D \\
L_{kin} &= \int d^3y (\partial_m \bar{a}_i \partial^m a_i + \partial_m \bar{b}_i \partial^m b_i) \\
L_{gauge} &= \int d^3y (v_{mn}^2 + \partial_m S \partial^m S) \\
L_D &= -r \int d^3y D.
\end{aligned} \tag{7}$$

The action of the chiral fields which depend on a three dimensional space is the same as those of the two dimensional action (1). The same occurs for the D-term. However, the content of the D=3, N=2 vector multiplet is different from its two-dimensional analog because there is only one scalar S . The bosonic potential of the D=3 action is also obtained after integrating out the auxiliary field D

$$U = \frac{e^2}{2} (\sum_i |a_i|^2 - |b_i|^2 - r)^2 + S^2 (\sum_i |a_i|^2 + |b_i|^2). \tag{8}$$

For $r \gg 0$ and demanding preservation of N=2 supersymmetry, we obtain an action for a supermembrane propagating on a manifold X , which has a symplectic quotient Z_+ . For $r \ll 0$, we obtain the action for a supermembrane propagating on a manifold Y which is birrationally equivalent to the manifold X and has a symplectic quotient Z_- .

We are not able to go from positive to negative r without encountering a sharp phase transition because there is no theta-angle to prevent the spectrum from becoming continuous for wave functions supported near $a_i = b_i = 0$ and $S \gg 0$. In order to obtain smooth phase transitions, we must couple the supermembrane to the 3-form antisymmetric tensor C which is part of the massless spectrum of D=11 supergravity. This coupling also insures k-symmetry of the supermembrane action [8, 12]. This is done by adding a term to the D=3 Lagrangian (7) whose bosonic sector is

$$m \int d^3y (\epsilon^{ijk} A_i \partial_j A_k + S^2). \quad (9)$$

This term is particular to three dimensions: it is a mass term for the worldvolume U(1) gauge multiplet (A_i, S) which preserves gauge symmetry as well as N=2 supersymmetry. As opposed to the topological coupling θ in (2) which is periodic, the coupling m takes values on \mathbf{Z} [14]² and it is the coupling of the membrane to the three form potential C .

Now we may consider the action for a supermembrane which has a rational curve on a manifold X which can be flopped to a rational curve of a manifold Y . For $r \gg 0$, we encounter the worldvolume action on a rational curve on a manifold X . For $r \ll 0$ we encounter the worldvolume action on a rational curve on a manifold Y which is birrationally equivalent to the manifold X . In order to go from X to Y we must pass through the point $r = 0$. At this point, if $m = 0$ we find wave functions supported near $a_i = b_i = 0$ and $S \gg 0$ which have a continuous spectrum and therefore a singularity at $r = 0$. However, if $m \neq 0$ such wave functions are absent because $S = 0$, and we are guaranteed a smooth phase transition in going from X to Y . The topological term in the two dimensional action is analogous to the topological term of the three dimensional action. Since the former yields stringy effects which cancel the continuous spectrum, the latter should also be thought as a membrany quantum effect. Thus, M-theory has smooth phase transitions like string

²I thank E. Witten for pointing this out to me.

theory when transversing a boundary of a Kahler cone provided quantum effects are included in the effective membrane action.

We see that different values of m lead to different physical situations. In particular, a membrane with $m = 0$ does not yield smooth transitions while a membrane with $m \neq 0$ do. Thus, the two membranes are different and yield different phases for M-theory.

4 Phase Transitions Between Geometric and Non-geometric Phases of Supermembranes

So far we have considered the phases of M-theory within the boundary of the extended Kahler cone (the set of Kahler cones which yields geometric phases) and on a local patch containing an inner boundary which does not belong to the boundary of the extended Kahler cone. We would now like to show that the D=3 Lagrangian indeed describes a membrane propagating on a Calabi-Yau manifold, and also analyze the transition of M-theory to a Landau-Ginzburg orbifold phase which, as explained in [1], is a cone of zero size, and therefore defines the boundary of the extended Kahler cone.

For simplicity, we will consider a Calabi-Yau manifold given by the quintic polynomial embedded on CP^4 . In order to define a membrane on this Calabi-Yau manifold, we must consider the action whose bosonic sector is³

$$\begin{aligned}
L &= L_{kin} + L_{gauge} + L_D + L_W \\
L_{kin} &= \int d^3y (\partial_m \bar{\phi}_i \partial^m \phi_i + \partial_m \bar{p} \partial^m p) \\
L_{gauge} &= \int d^3y (v_{mn}^2 + \partial_m S \partial^m S) + m \int d^3y (\epsilon^{ijk} A_i \partial_j A_k + S^2) \\
L_D &= -r \int d^3y D
\end{aligned}$$

³Here we also use the conventions of [11]

$$L_W = \int d^3y F_i \frac{\partial W}{\partial \phi_i} + F_p \frac{\partial W}{\partial p}. \quad (10)$$

The chiral fields are $\Phi_i ; i = 1, \dots, 5$, and P , their bosonic scalars are ϕ_i and p respectively, and their auxiliary fields are F_i and F_p respectively. The chiral fields Φ_i and P will have charge $+1$ and -1 respectively with respect to the $U(1)$ field. The new structure in the Lagrangian is the term L_W which is the potential of the model. It has the form

$$W = pQ(\phi) \quad (11)$$

where Q is a homogeneous degree five polynomial. It is also transverse, so that its first derivatives with respect to the ϕ 's vanish simultaneously only when all ϕ 's vanish. Integrating out the auxiliary fields F_i , F_p and D , we obtain the bosonic potential

$$U = \frac{e^2}{2} (\sum_i |\phi_i|^2 - |p|^2 - r)^2 + S^2 (\sum_i |\phi_i|^2 + |p|^2) + |Q(\phi)|^2 + |p|^2 \sum_i \left| \frac{\partial Q}{\partial \phi_i} \right|^2 + mS^2. \quad (12)$$

For $r \gg 0$ we see that the ϕ 's cannot all vanish. Then, by the transversality of the polynomial Q , p must vanish. The vanishing of the first term in the bosonic potential yields an S^9 which after modding out by the $U(1)$ gauge symmetry becomes a CP^4 with Kahler class proportional to r . The condition that Q vanish in order to minimize the potential, defines an embedding of the quintic polynomial Q on CP^4 which yields a Calabi-Yau manifold: the quintic. As in string theory, the scalar S and the gauge field both become massive. Thus the effective theory for large r is a supermembrane propagating on the quintic. This is to be compared with the phase analysis done for strings in [11]. They both have the same behavior for large r , and dimensional reduction of the effective action (10) on $S^1 \times X$ yields a string propagating on X .

The case in which $r \ll 0$ is also similar to that of strings. There, p cannot vanish and transversality of Q implies that all ϕ 's must vanish; the $U(1)$ vector multiplet becomes massive. The massless theory is a Landau-Ginzburg orbifold (LGO) for the

membrane. It is not possible to make a statement of the infrared and ultraviolet behavior because the action is nonrenormalizable.

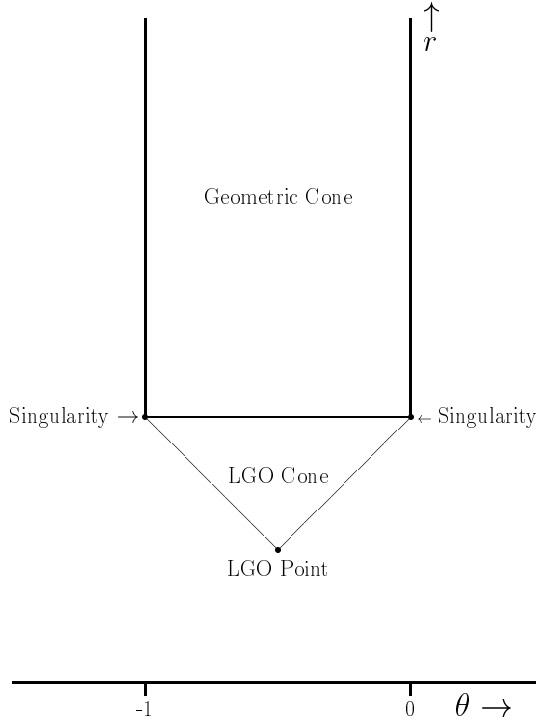


Figure 1. The Phases of Strings

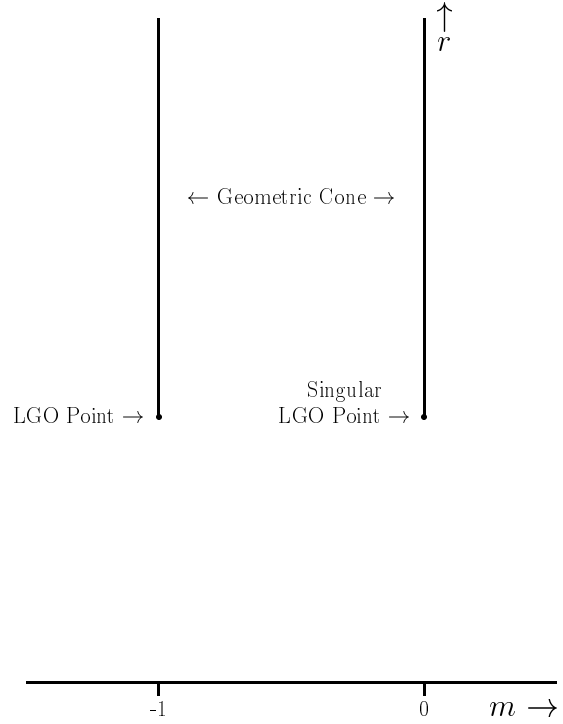


Figure 2. The Phases of Membranes

So far, we have been working with linear sigma model coordinates. In string theory, for positive r the cone in special coordinates is of the same size as in linear sigma model coordinates because instanton effects are small. However, for negative r , where instanton effects are large, the LGO cone in special coordinates is squashed to a thickness of order α' . This is shown in Figure 1, where the LGO phase is the triangle formed by the LGO point and the two singularities. The Kahler class of the LGO cone in type IIA string theory can be related to the Kahler class of the LGO cone in supermembrane theory and the radius R of compactification of the eleventh

dimension [1]

$$K_M = \frac{K_{II}}{T^{\frac{1}{3}}R} \quad (13)$$

where T is the membrane tension. This relation implies that in the limit in which $R \rightarrow \infty$, or equivalently $\alpha' \rightarrow 0$, the Kahler cone of the LGO phase of the supermembrane, is squash to the boundary of the extended Kahler cone. The moduli space of the supermembrane is shown in Figure 2.

For $m \neq 0$, the LGO phase forms a boundary of the extended Kahler cone of geometric phases. The LGO point which lives on this boundary is continuously connected to the extended Kahler cone of geometric phases and has a discrete spectrum, thus making it a well defined phase in M-theory. Notice that once more, different values of m lead to different physical situations. In particular, a membrane with $m = 0$ has a singular LGO point with continuous spectrum while a membrane with $m \neq 0$ has an LGO phase with discrete spectrum. Again, these two membranes have different phases and lead to different physics.

5 Axionic Potential of the Membrane on a 3-Fold

We have seen in the two previous sections that the moduli obtained by wrapping the three-form potential C about a three-cycle take integral values. We have also seen that the physics for different non zero values of these moduli is the same and that the physics for non zero and zero values of the moduli is different. For non zero values the membrane has smooth transitions between Kahler cones and also has smooth transitions to well defined Landau Ginzburg phases. For zero values of these moduli we find that the Landau Ginzburg phases is singular and the transition between different Kahler cones is sharp.

In this section we will extend the properties of these moduli to those moduli which are also obtained by wrapping the three form potential C about a three-cycle which can be written as a product of a two-cycle and a one-cycle. This mild assumption,

which is based on the fact that there is no underlying reason why the membrane should distinguish between a simply connected and a non simply connected cycle, will allow us to perform a dimensional reduction of 11 dimensional supergravity to 10 dimensional type IIA supergravity and relate the moduli of the C field to the axion of type IIA supergravity obtained by wrapping its two-form potential B about a two cycle. This relation between the moduli of the C field, which we will refer to as M-theory axion a_M , and the usual axion which we will refer to as type IIA axion a_S will serve to establish one property: the existence of a potential for the axion in the strong coupling limit which does not depend on instanton contributions.

We first show that the axion a_M is periodic by closely following the analysis of Rhom and Witten [13] to show the periodic properties of a_S . Note that the term

$$I = \int_W d\Sigma^{NMPQ} G_{NMPQ} \quad (14)$$

present in the supermembrane action does not need to be single-valued. Here W is a closed four manifold and

$$G = dC. \quad (15)$$

Rather, it is e^{iI} which must satisfy this property. This implies that $I = 2\pi n$ where $n \in \mathbf{Z}$

We now define a_M to be

$$a_M(x^\mu) = \int_T d\Sigma^{MNP} C_{MNP} \quad (16)$$

where the integral is over the closed three manifold. Then, in circling a loop γ at fixed transverse distance from a membrane which we will parametrize by the angular coordinate ϕ , the change in a_M is

$$\delta a_M = \int_0^{2\pi} d\phi \frac{da_M}{d\phi} = \int_{\gamma \times T} dC = \int_{\gamma \times T} G. \quad (17)$$

Since $\gamma \times T$ is a closed four manifold we find that

$$\delta a_M = 2\pi n \quad (18)$$

and therefore a_M is a periodic variable.

Having established the periodicity of a_M we continue by considering 11 dimensional supergravity to show the existence of an axionic potential. The bosonic sector of the action is

$$\int d^{11}x (\sqrt{-g}(R + G^2) + G \wedge G \wedge C) \quad (19)$$

We will dimensionally reduce this action to four dimensions. The internal space used to reach four dimensions is $X \times S^1$ where X is a Calabi-Yau threefold. A scalar field a will be an axion only if it satisfies that it is periodic and if it exhibits a coupling of the form

$$\int d^4x a F \tilde{F} \quad (20)$$

where F is a two-form field strength and \tilde{F} its dual. A term of the form (20) can only be obtained from the topological term in (19). The two-form field strength is obtained by wrapping H about a two-cycle found in X

$$H_{\mu\nu mn} = F_{\mu\nu}(x^\rho) \wedge b_{mn}^I(y^s) \quad (21)$$

where μ, ν label space time coordinates, m, n label internal coordinates of X and b^I is a two-form on X . We have two choices to obtain a scalar from the C field. The first and less interesting is by wrapping the C field about a three-cycle of X , the second and more interesting for our discussion is to wrap C about a two-cycle of X and the one-cycle of S^1 which will lie in the 11th dimension

$$C_{mn11} = a_M^I(x^\mu) b_{mn}^I(y^s, x^{11}) \wedge b_{11}(y^s, x^{11}) \quad (22)$$

where a_M will be the M-theory axion and b_{11} is the one-form on S^1 . b^I is again a two-form on X .

With this ansatz, the GGC term in (19) takes the form

$$\int d^4x a_M^K F^J \tilde{F}^I \int d^6y dx^{11} b_{11} \wedge b^I \wedge b^J \wedge b^K \quad (23)$$

Thus the scalar a_M^K is the axion of M-theory. As we saw in the previous sections, a_M^K takes integral values. This is not the case of the axion of type IIA supergravity.

The type IIA axion is obtained from the topological term of type IIA supergravity

$$\int d^{10}x \, G \wedge G \wedge B \quad (24)$$

where G is the four-form field strength of type IIA supergravity and B is the two for potential of type IIA supergravity. Substituting into (24) the ansatz

$$\begin{aligned} G_{\mu\nu mn} &= F_{\mu\nu}^I(x^\mu) \wedge b_{mn}^I(y^s) \\ B_{mn} &= a_S^I(x^\mu) b_{mn}^I(y^s) \end{aligned} \quad (25)$$

where b^I is a two-form on X we obtain

$$\int d^4x \, a_S^K F^J \tilde{F}^I \int d^6y \, b^I \wedge b^J \wedge b^K \quad (26)$$

A close look at (26) reveals that a_S has an axion like coupling. Furthermore, as we have seen in this section, a_M is periodic and so is a_S [13]. Thus, both a_M and a_S behave like axions which also depend on the type of compactification and are therefore model dependent axions. Another property they share is that a_M^I is the strong coupling of a_S^I . Indeed, in the limit that the S^1 parametrized by x^{11} is very large, we are in the M-theory regime which is the strong coupling limit of type IIA supergravity. In this strong coupling limit, a_M is the axion. When the S^1 is very small, M-theory collapses to type IIA supergravity and a_M collapses to a_S . This follows from the fact that for very small S^1 we can take the fields tangent to X to be independent of x^{11} and then (23) must collapse to (26). This can only happen provided we identify a_M with a_S in this limit.

This identification between a_M and a_S along with what we have learned in the previous section about a_M imply that a potential for the axion is generated in the strong coupling limit. In the strong coupling regime, a_M can only take integral values. We have further shown that while $a_M \neq 0$ and $a_M = 0$ lead to different physical

behavior of the membrane, we cannot really distinguish between the different possible values of a_M which are different from zero. Thus we can use the identification

$$a_M \sim a_M + 1 \tag{27}$$

when $a_M \neq 0$. Therefore, we have two possible values: $a_M = 0, 1$.

Now consider the situation in which we start in the weak limit of M-theory (type IIA supergravity). In this case, the axion is a_S and it can take any value between 0 and 1. As we increase the coupling constant and reach the M-theory regime, we find that the value of a_S is now that of a_M and therefore is confined to be either 0 or 1. This means that we start with a flat potential in the weak coupling limit and that as we move to the strong coupling limit a potential (perhaps of sinusoidal shape) develops and forces a_S to take two possible values.

From the physical point of view we would like the axion to be very small and therefore, the value of $a_S = 0$ should be picked by nature to the value of $a_S = 1$. There is not apparent reason to distinguish between one or the other in the model we have just analyzed, but we are tempted to say that perhaps the presence of smooth phase transitions and LGO phases may play a role in destabilizing the second vacuum. In any case, the existence of a potential implies that the axion picks up a mass. With our present knowledge of supermembranes, we are not able to give an order of magnitude for the axion mass and therefore we are not able to make any statement about the applicability of this axion potential to strong CP problem.

6 The Winding Conjecture

As explained in the introduction, we need a mechanism that will allow us to treat multiple windings of membranes as multiple particle states while treating multiple windings of the string as single particle states. From the discussion of sections 3 and 4, we have learned that the coupling of the string to the two-form potential is periodic and the coupling of the membrane to the three-form potential is periodic

and discrete. The difference between these two couplings can be used to explain why multiple windings of the membrane can be interpreted as multiple particle states while multiple windings of the string can be interpreted as single particle states.

6.1 Model Dependent Degrees of Freedom in the Gauged Linear Sigma Model

A string propagating on a Calabi-Yau threefold does not have gauge degrees of freedom which are model dependent. A membrane can wrap about a two-cycle on the manifold and can thus have model dependent gauge degrees of freedom. For a compactification T^2 , both the membrane and the string can have model dependent gauge degrees of freedom associated to the B-field and C-field respectively. For the string they arise from wrapping the B-field about the one cycles of T^2 while for the membrane they arise from wrapping the C-field about the two cycle of T^2 . The question is how these degrees of freedom arise in the string gauged linear sigma model. Clearly, the expression

$$\int d^2y \partial_i \phi^n \partial_j X^\mu \epsilon^{ij} B_{\mu n} \quad (28)$$

is not gauge invariant, and it is thus excluded from the list of possible terms which can be added to the string gauged linear sigma model. However, we must remember that the string is 10 dimensional and not two dimensional. Thus the pull back of the term

$$\int d^2y \epsilon^{ij} \partial_j A_i = \int d^2y \epsilon^{ij} B_{ij} \quad (29)$$

is not

$$\int d^2y \epsilon^{ij} \partial_j \phi^n \partial_i \phi^m B_{mn} \quad (30)$$

which is the pull back to a conformally anomalous two ($=\dim(T^2)$) dimensional compact target space with coordinates labeled by index n, m . Rather, for a target space

$T^2 \times R^8$ (29) takes the form

$$\int d^2 y \epsilon^{ij} (\partial_j \phi^n \partial_i \phi^m B_{mn} + \partial_j X^\mu \partial_i X^\nu B_{\mu\nu} + \partial_j \phi^n \partial_i X^\mu B_{\mu n}), \quad (31)$$

which is the embedding of a string in a 10 dimensional space. Thus, the model dependent gauge degrees of freedom associated to the B-field are found in the topological term of the string gauged linear sigma model.

We can make a similar treatment of the topological term

$$\int d^3 y \epsilon^{ijk} \partial_j A_i A_k = \int d^3 y \epsilon^{ijk} C_{ijk} \quad (32)$$

found in the membrane gauged linear sigma model. The pull back of (32) must include all the fields in the 11 dimensional space. Thus, the pullback of (32) for a $T^2 \times R^9$ target space takes the form

$$\int d^3 y \epsilon^{ijk} (\partial_i \phi^m \partial_j \phi^n \partial_k X^\mu C_{mn\mu} + \partial_i \phi^n \partial_j X^\mu \partial_k X^\nu C_{n\mu\nu} + \partial_i X^\mu \partial_j X^\nu \partial_k X^\rho C_{\mu\nu\rho}). \quad (33)$$

Otherwise, dimensional reduction of (33) would not be consistent with (31) in the limit that $T^2 \rightarrow S^1 \times R$.

6.2 Evidence for the Conjecture

We continue by studying the different behaviors of the Dirac action for a membrane wrapped once about an S^1 and a membrane wrapped n times about the same S^1 . The induced metric for the 11 dimensional background must be expressed in terms of 10 dimensional backgrounds in order to carry out the dimensional reduction [6]. This decomposition leads to an induced worldvolume metric of the form

$$\hat{g}_{\hat{i}\hat{j}} = \Phi^{-2/3} \begin{pmatrix} g_{ij} + \Phi^2 A_i A_j & \Phi^2 A_i \\ \Phi^2 A_j & \Phi^2 \end{pmatrix}. \quad (34)$$

For this decomposition and a membrane wrapped once about the S^1 we find that $\sqrt{\hat{g}} = \sqrt{g}$. In the case that the membrane is wound n times about S^1 we have instead $\sqrt{\hat{g}} = n\sqrt{g}$. This is expected because $\sqrt{\hat{g}}$ measures the worldvolume, and by wrapping

the membrane n times about S^1 , the worldvolume is increased by a factor n . We can absorb this factor by simply redefining the worldvolume metric since the membrane action is not Weyl invariant.

We could be tempted to interpret the n in the worldvolume factor as a rescaling of the membrane tension. That is, *a priori* we could use the argument that a membrane with n units of worldvolume and tension T has the same energy as a membrane with a single unit of worldvolume and tension nT to argue that they lead to the same physics. However, this statement is not true. The dimensional reduction of these two membranes lead to strings with different physics. The first case leads to a string with tension proportional to T and n units of worldsheet. The second case leads to a string with tension proportional to nT and unit worldsheet. In the first case we can use the Weyl symmetry to get a string with unit worldsheet and tension T . This string has the same worldsheet as the string in the second case but its tension is n times larger than the first string. Therefore, we cannot interpret the n in the worldvolume factor as a rescaling of the membrane tension.

From the paragraph above, we arrive to the conclusion that *the string tension is not rescaled by the winding number of the membrane about the S^1 and therefore a membrane wrapped n times about S^1 does not lead to a rescaling of the string tension, rather, it leads to a redefinition of the worldvolume metric.*

We now take a look at the WZ term for the membrane. The three form potential, C , appears in the membrane action in the term

$$m \int d^3y \epsilon^{ijk} \partial_i X^\mu \partial_j X^\nu \partial_k \phi^{11} C_{11\mu\nu} \quad (35)$$

where $m \in \mathbf{Z}$. It is the pull back of the topological term (9). We consider the situation in which the membrane is wrapped n times about the one-cycle of S^1 . This is done by using the reparametrization

$$\phi^{11} = ny^3 \quad (36)$$

where y^3 , a coordinate on the world volume, along with ϕ^{11} will be dimensionally reduced. Before doing any dimensional reduction we are already able to make a statement. Substitution of (36) into (35) yields

$$nm \int d^3y \epsilon^{ij3} \partial_i X^\mu \partial_j X^\nu C_{11\mu\nu} \quad (37)$$

The coupling of the C field after using reparametrization invariance is rescaled

$$m \rightarrow nm. \quad (38)$$

However, m is periodic. Thus

$$m \sim m + 1 \quad (39)$$

means that

$$mn \sim m \quad (40)$$

and therefore, the axionic charge of the membrane is not renormalized by the winding number of the membrane about S^1 .

This means that the membrane wrapping n times about the S^1 does not lead to a rescaling of the axionic charge. After dimensional reduction, the string action obtained has unit axionic charge regardless of how many times the membrane wraps around the S^1 .

It is the integral nature of the coupling m which has prevented the axionic charge from being rescaled. This is not the case for the string because the coupling of the WZ term in the string action is not integral. Then, the WZ term for the string

$$\theta \int d^2y \epsilon^{ij} \partial_i X^\mu \partial_j \phi^{10} B_{10\mu} \quad (41)$$

will have its coupling θ rescaled after the reparametrization

$$\phi^{10} = ny^2 \quad (42)$$

Substitution of (42) into (41) gives

$$n\theta \int dy \epsilon^{i2} \partial_i X^\mu B_{10\mu}. \quad (43)$$

Therefore, the string coupling gets renormalized by the winding number of the string about a one-cycle

$$\theta \rightarrow n\theta. \quad (44)$$

For generic values of θ there is no identification

$$\theta \sim n\theta \quad (45)$$

because θ is not an integer. Thus the “axionic” charge of space time particles depend on the number of times the string is wound about S^1 , as expected.

These contrasting properties between m and θ allow us to resolve the problem of winding conjecture to a membrane wrapped about T^2 . Let us start with a membrane wrapped n_1 times about the first one-cycle of T^2 and wrapped n_2 times about the second one-cycle of T^2 . We first perform a dimensional reduction of the WZ term for the membrane about the first one-cycle. This yields a WZ term for the string

$$m \int d^2y \epsilon^{ij3} \partial_i X^\mu \partial_j X^\nu C_{11\mu\nu} \quad (46)$$

where we have used property (40). We must now use the ansatz

$$C_{11\mu\nu} = a(x^\mu) B_{\mu\nu} \quad (47)$$

which takes the expression (46) to the form

$$\theta m \int d^2y \epsilon^{ij} \partial_i X^\mu \partial_j X^\nu B_{\mu\nu} \quad (48)$$

where θ is the expectation value of $a(x^\mu)$ which has a periodic nature

$$\theta \sim \theta + 1 \quad (49)$$

and therefore

$$m\theta \sim \theta' \in [0, 2\pi]. \quad (50)$$

Thus, expression (46) takes the final form

$$\theta' \int d^2y \epsilon^{ij} \partial_i X^\mu \partial_j X^\nu B_{\mu\nu}. \quad (51)$$

This expression is the WZ term for a string propagating in a 10 dimensional background. Since the topology of the space is $R^9 \times S^1$ we find that the WZ term is

$$\theta' \int d^2y \epsilon^{ij} \partial_i X^\mu \partial_j \phi^l B_{\mu l}. \quad (52)$$

As explained above, dimensional reduction of the string WZ term to a superparticle action will depend on how many times the string is wound about the S^1 . Thus, in carrying out the a second dimensional reduction about the second one-cycle in order to obtain a particle, we find that the axionic charge is renormalized by the winding of the string and therefore that multiple winding of the string lead to different particle states, in agreement with [5].

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